Fundamental physics of anisotropic medium

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Tensors

- generalization of vector definition
- general theory in n-dimensional space
- theory application for the physical properties of anisotropic materials (especially for single-crystals) → n=3
- tensor's order q
    - ➢ number of components $p=n^q$
    - – scalar (zeroth order tensor) → p=1
    - – vector (first order tensor) → p=3
    - – second order tensor → p=9
    - – third order tensor → p=27
    - – fourth order tensor → p=81

Tensors

- vector: $T'_i = a_{ij} T_j$
  - transforms as the coordinate
- second order tensor: $T'_{ij} = a_{ik} a_{jl} T_{kl}$
  - transforms as the product of two coordinates
- third order tensor: $T'_{ijk} = a_{il} a_{jm} a_{kn} T_{lmn}$
  - transforms as the product of three coordinates
- fourth order tensor: $T'_{ijkl} = a_{im} a_{jn} a_{ko} a_{lp} T_{mnop}$
  - transforms as the product of four coordinates

Tensors

- symmetrical second order tensor

  $T_{ij}=T_{ji}$ → 6 components

- antisymmetrical second order tensor

  $T_{ij}=-T_{ji}$ → 3 components (axial vector!)

- symmetry of higher order tensors can be given by the combinations of two or more indexes (see special examples)

Examples of the physical quantities with tensor character

- first order tensor (vector)
  - physical "state" quantities (electric field E_i, electric current density j_i)
  - material properties (pyroelectric coefficient p_i)
- second order tensor
  - physical "state" quantities (mechanical stress σ_{ij}, strain ϵ_{ij})
  - material properties (electrical conductivity γ_{ij}, thermal expansion α_{ij}, dielectric permittivity ε_{ij})
- third order tensor
  - material properties only (piezoelectric coefficient d_{ijk})
    - by definition $d_{ijk}=d_{ikj}$ → 18 components
- fourth order tensor
  - material properties only (elastic compliance s_{ijkl})
    - by definition $s_{ijkl}=s_{jikl}=s_{ijlk}=s_{jilk}$ → 21 components

Symmetry elements and symmetry operations

Could be described by the mathematical group theory.

- rotation axes (twofold, threefold, fourfold and sixfold) → rotation
- inverse rotation axes (threefold, fourfold and sixfold) → inverse rotation
- symmetry plane → mirror
- center of symmetry → center inversion

Crystallographic systems and symmetry point groups

Seven crystallographic systems contain 32 point groups of symmetry.

- triclinic – center of symmetry, 2 point groups
- monoclinic – 1 twofold rotation or inverse rotation axis, 3 groups
- orthorhombic – 3 mutually perpendicular twofold rotation or inverse rotation axes, 3 groups
- tetragonal – 1 fourfold rotation or inverse rotation axis, 7 groups
- trigonal (rhombohedral) – 1 threefold rotation or inverse rotation axis, 5 groups
- hexagonal – 1 hexagonal rotation or inverse rotation axis, 7 groups
- cubic – 4 threefold rotation or inverse rotation axes as cube’s body diagonals, 5 groups

Neumann's principle

- Set of symmetry elements for the material characteristics (physical property) must include all symmetry elements of the medium (point group of single-crystal's symmetry), i.e. symmetry of the material properties must be the same or higher than the symmetry of the medium