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Optical components for polarimetry

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Outline

• Complex index of refraction

• Polarizers

• Retarders
The index of refraction is actually a complex quantity:

\[ m = n - ik \]

- **Real part**
  - Optical path length, refraction: speed of light depends on media
  - **Birefringence**: speed of light also depends on P

- **Imaginary part**
  - Absorption, attenuation, extinction: depends on media
  - **Dichroism/diattenuation**: also depends on P
Polarizers absorb one component of the polarization but not the other.

The input is natural light, the output is polarized light (linear, circular, elliptical). They work by dichroism, birefringence, reflection, or scattering.
Wire-grid polarizers (I) [dichroism]

• Mainly used in the IR and longer wavelengths
• Grid of parallel conducting wires with a spacing comparable to the wavelength of observation
• Electric field vector parallel to the wires is attenuated because of currents induced in the wires
Wide-grid polarizers (II)
[dichroism]
Dichroic crystals absorb one polarization state over the other one.

Example: tourmaline.
Polaroids
[dichroism]

Made by heating and stretching a sheet of PVA laminated to a supporting sheet of cellulose acetate treated with iodine solution (H-type polaroid). Invented in 1928.
Crystal polarizers (I)
[birefringence]

• Optically anisotropic crystals

• Mechanical model:
  • the crystal is anisotropic, which means that the electrons are bound with different ‘springs’ depending on the orientation
  • different ‘spring constants’ gives different propagation speeds, therefore different indices of refraction, therefore 2 output beams
Crystal polarizers (II)
[birefringence]

The 2 output beams are polarized (orthogonally).

- isotropic crystal (sodium chloride)
- anisotropic crystal (calcite)
Crystal polarizers (IV) [birefringence]

- Crystal polarizers used as:
  - Beam displacers,
  - Beam splitters,
  - Polarizers,
  - Analyzers, ...
- Examples: Nicol prism, Glan-Thomson polarizer, Glan or Glan-Foucault prism, Wollaston prism, Thin-film polarizer, ...
Mueller matrices of polarizers (I)

• (Ideal) linear polarizer at angle $\chi$:

$$
\begin{pmatrix}
1 & \cos 2\chi & \sin 2\chi & 0 \\
\cos 2\chi & \cos^2 2\chi & \sin 2\chi \cos 2\chi & 0 \\
\sin 2\chi & \sin 2\chi \cos 2\chi & \sin^2 2\chi & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
$$
## Mueller matrices of polarizers (II)

### Optical components for polarimetry

**Linear (±Q) polarizer at 0°:**
\[
\begin{pmatrix}
1 & \pm 1 & 0 & 0 \\
\pm 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

**Linear (±U) polarizer at 0°:**
\[
\begin{pmatrix}
1 & 0 & \pm 1 & 0 \\
0 & 0 & 0 & 0 \\
\pm 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

**Circular (±V) polarizer at 0°:**
\[
\begin{pmatrix}
1 & 0 & 0 & \pm 1 \\
0 & 0 & 0 & 0 \\
\pm 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

- 0.5
Mueller calculus with a polarizer

Input light: unpolarized --- output light: polarized

\[
\begin{bmatrix}
I' \\
Q' \\
U' \\
V'
\end{bmatrix}
= 0.5 \begin{bmatrix}
1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
I \\
0 \\
0 \\
0
\end{bmatrix}
= 0.5 \begin{bmatrix}
I \\
0 \\
0 \\
-1
\end{bmatrix}
\]

Total output intensity: 0.5 I
Retarders

- In retarders, one polarization gets ‘retarded’, or delayed, with respect to the other one. There is a final phase difference between the 2 components of the polarization. Therefore, the polarization is changed.

- Most retarders are based on birefringent materials (quartz, mica, polymers) that have different indices of refraction depending on the polarization of the incoming light.
Half-Wave plate (I)

• Retardation of $\frac{1}{2}$ wave or $180^\circ$ for one of the polarizations.

• Used to flip the linear polarization or change the handedness of circular polarization.
Half-Wave plate (II)
Quarter-Wave plate (I)

• Retardation of ¼ wave or 90° for one of the polarizations

• Used to convert linear polarization to elliptical.
Quarter-Wave plate (II)

- Special case: incoming light polarized at 45º with respect to the retarder’s axis

- Conversion from linear
Mueller matrix of retarders (I)

- Retarder of retardance $\tau$ and position angle $\psi$:

$$
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & G + H \cos 4\psi & H \sin 4\psi & -\sin \tau \sin 2\psi \\
0 & H \sin 4\psi & G - H \cos 4\psi & \sin \tau \cos 2\psi \\
0 & \sin \tau \sin 2\psi & -\sin \tau \cos 2\psi & \cos \tau
\end{pmatrix}
$$

with: $G = \frac{1}{2}(1 + \cos \tau)$ and $H = \frac{1}{2}(1 - \cos \tau)$
Mueller matrix of retarders (II)

- Half-wave oriented at $0^\circ$ or $90^\circ$

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

- Half-wave oriented at $\pm 45^\circ$

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]
Mueller matrix of retarders (III)

• Quarter-wave oriented at \(0^\circ\)

\[
k = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0
\end{pmatrix}
\]

• Quarter-wave oriented at \(\pm 45^\circ\)

\[
k = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mp 1 \\
0 & 0 & 1 & 0 \\
0 & \pm 1 & 0 & 0
\end{pmatrix}
\]
Mueller calculus with a retarder

- Input light linear polarized (Q=1)
- Quarter-wave at +45°
- Output light circularly polarized (V=1)

\[
\begin{pmatrix}
I' \\
Q' \\
U' \\
V'
\end{pmatrix}
= k
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 & 0 \\
0 & +1 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix}
= k
\begin{pmatrix}
1 \\
0 \\
0 \\
1
\end{pmatrix}
\]
(Back to polarizers, briefly)

Circular polarizers

- Input light: unpolarized  ---  Output light: circularly polarized
- Made of a linear polarizer glued to a quarter-wave plate oriented at 45º with respect to one another.
Achromatic retarders (I)

- Retardation depends on wavelength
- Achromatic retarders: made of 2 different materials with opposite variations of index of refraction as a function of wavelength
- Pancharatnam achromatic retarders: made of 3 identical plates rotated w/r one another
- Superachromatic retarders: 3 pairs of quartz and MgF$_2$ plates
Achromatic retarders (II)

\[ \tau = 140-220^\circ \]
not very achromatic!

\[ \tau = 177-183^\circ \]
much better!

Fig. 1. Wavelength dependence of retardance for a half-wave plate of magnesium fluoride and quartz.

Fig. 2. Wavelength dependence of (a) the position angle of the equivalent optical axis, and (b) the retardance for the Pancharatnam achromatic half-wave plate of magnesium fluoride and quartz.
Retardation on total internal reflection

- Total internal reflection produces retardation (phase shift)

- In this case, retardation is very achromatic since it only depends on the refractive index

- Application: Fresnel rhombs
Fresnel rhombs

- Quarter-wave and half-wave rhombs are achieved with 2 or 4 reflections
Other retarders

- Soleil-Babinet: variable retardation to better than 0.01 waves
- Nematic liquid crystals... Liquid crystal variable retarders... Ferroelectric liquid crystals... Piezo-elastic modulators... Pockels and Kerr cells...
Part IV: Polarimeters

- Polaroid-type polarimeters
- Dual-beam polarimeters
Polaroid-type polarimeter
for linear polarimetry (I)

• Use a linear polarizer (polaroid) to measure linear polarization ...
  [another cool applet]
  Location: http://www.colorado.edu/physics/2000/applets/lens.html

• Polarization percentage and position angle:

\[
P = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]

\[
\Theta = \Theta(I = I_{\text{max}})
\]
Polaroid-type polarimeter
for linear polarimetry (II)

- Move the polaroid to **2 positions**, 0° and 45° (to measure Q, then U)

- Advantages:
- Disadvantages:
- Other disadvantages: non-simultaneous measurements, cross-talk...
Polaroid-type polarimeter for circular polarimetry

- Polaroids are not sensitive to circular polarization, so convert circular polarization to linear first, by using a quarter-wave plate
- Polarimeter now uses a quarter-wave plate and a polaroid
- Same disadvantages as before
Dual-beam polarimeters

Principle

• Instead of cutting out one polarization and keeping the other one (polaroid), split the 2 polarization states and keep them both
• Use a Wollaston prism as an analyzer
• Disadvantages: need 2 detectors (PMTs, APDs) or an array; end up with 2 ‘pixels’ with different gain
• Solution: rotate the Wollaston or keep it fixed and use a half-wave plate to switch the 2 beams
Dual-beam polarimeters
Switching beams

- Unpolarized light: two beams have identical intensities whatever the prism’s position if the 2 pixels have the same gain

- To compensate different gains, switch the 2 beams and average the 2 measurements

\[ \Delta \sum \]
Dual-beam polarimeters
Switching beams by rotating the prism

rotate by 180°
Dual-beam polarimeters

Switching beams using a $\frac{1}{2}$ wave plate
A real circular polarimeter
Semel, Donati, Rees (1993)

Quarter-wave plate, rotated at -45º and +45º

Analyser: double calcite crystal
A real circular polarimeter free from gain (g) and atmospheric transmission (∝) variation effects

• First measurement with quarter-wave plate at -45°, signal in the (r)ight and (l)eft beams:
  \[ S^l_1, \quad S^r_1 \]

• Second measurement with quarter-wave plate at +45°, signal in the (r)ight and (l)eft beams:
  \[ S^l_2, \quad S^r_2 \]

• Measurements of the signals:

\[
\begin{align*}
S^l_1 &= g^l_1 \alpha_1 (I_1 + V_1) \\
S^r_1 &= g^r_1 \alpha_1 (I_1 - V_1) \\
S^l_2 &= g^l_2 \alpha_2 (I_2 + V_2) \\
S^r_2 &= g^r_2 \alpha_2 (I_2 - V_2)
\end{align*}
\]
A real circular polarimeter free from gain and atmospheric transmission variation effects

• Build a ratio of measured signals which is free of gain and variable atmospheric transmission effects:

\[
F = \frac{1}{4} \left( \frac{S_1^l S_2^r}{S_2^l S_1^r} - 1 \right) = \frac{1}{2} \frac{I_2 V_1 + I_1 V_2}{I_1 I_2 - I_2 V_1 - I_1 V_2 + V_1 V_2}
\]

\[
F \approx \frac{1}{2} \left( \frac{V_1}{I_1} + \frac{V_2}{I_2} \right) \quad \text{for } V \ll 1
\]

average of the 2 measurements
Polarimeters - Summary

- **2 types:**
  - polaroid-type: easy to make but ½ light is lost, and affected by variable atmospheric transmission
  - dual-beam type: no light lost but affected by gain differences and variable transmission problems

- **Linear polarimetry:**
  - analyzer, rotatable
  - analyzer + half-wave plate

- **Circular polarimetry:**
  - analyzer + quarter-wave plate

- **Optical components for polarimetry**
  - 2 positions minimum
  - 1 position minimum
Part V: ESPaDOnS

Optical components of the polarimeter part:

- **Wollaston prism**: analyses the polarization and separates the 2 (linear!) orthogonal polarization states

- **Retarders, 3 Fresnel rhombs**:
  - Two half-wave plates to switch the beams around
  - Quarter-wave plate to do circular polarimetry
ESPaDOnS: circular polarimetry

• Fixed quarter-wave rhomb
• Rotating bottom half-wave, at 22.5º increments
• Top half-wave rotates continuously at about 1Hz to average out linear polarization when measuring circular polarization
ESPaDOnS: circular polarimetry of circular polarization

- Analyzer
- Half-wave
- 22.5° positions
- Flips polarization
- Gain, transmission
- Quarter-wave
- Fixed
- Circular to linear
ESPaDOnS: circular polarimetry of (unwanted) linear polarization

- Analyzer
- Circular part goes through not analyzed and adds same intensities to both beams
- Linear part is analyzed!

- Half-wave
  - 22.5° positions
  - Gain, transmission

- Quarter-wave
  - Fixed
  - Linear to elliptical

- Add a rotating half-wave to “spread out” the unwanted signal
ESPaDOnS: linear polarimetry

- Half-Wave rhombs positioned at 22.5° increments
- Quarter-Wave fixed
ESPaDOnS: linear polarimetry

- Half-Wave rhombs positioned as 22.5° increments
  - First position gives Q
  - Second position gives U
  - Switch beams for gain and atmosphere effects
- Quarter-Wave fixed
ESPaDOnS - Summary

- ESPaDOnS can do linear and circular polarimetry (quarter-wave plate)
- Beams are switched around to do the measurements, compensate for gain and atmospheric effects
- Fresnel rhombes are very achromatic
Used sources with thanks

N. Manset, CFHT

www.cfht.hawaii.edu/.../PolarizationLightIntro.ppt

E. Hecht

Optics - Undergraduate textbooks
Applets

Malus law
http://www.colorado.edu/physics/2000/applets/lens.html
Polarization of light, elipsometry - principle Malus law
http://physics-animations.com/Physics/English/optics.htm
Nematic crystals
http://www.colorado.edu/physics/2000/applets/nem2.html
LCD display
http://www.colorado.edu/physics/2000/applets/calc.html
Linear polarization
http://www.aldebaran.cz/animace/em_plane.gif
Elliptical polarization
http://www.aldebaran.cz/animace/em_elliptical.gif
Circular polarization
http://www.aldebaran.cz/animace/em_circular.gif
EM wave
http://www.aldebaran.cz/animace/em_stwave.gif
THANK YOU FOR YOUR ATTENTION