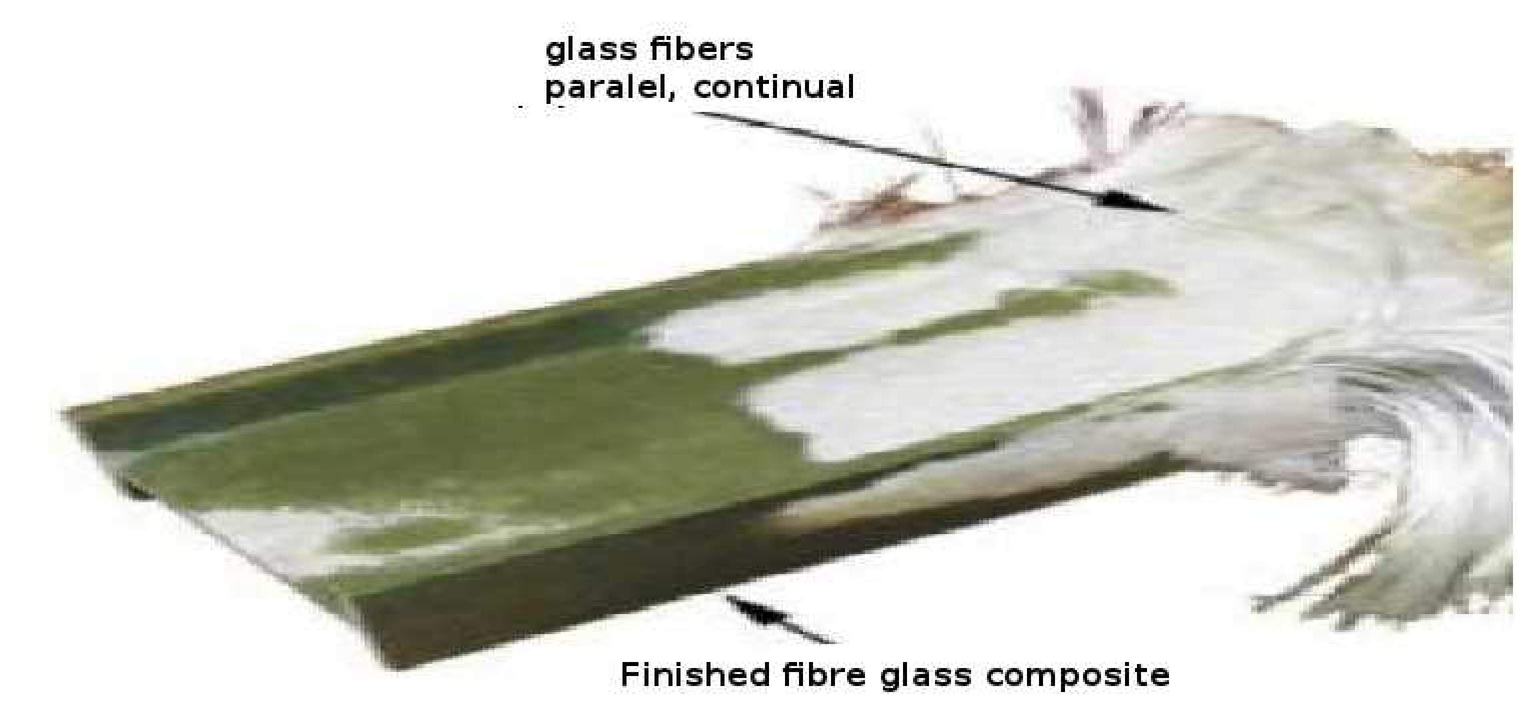
Voigt's model of composite



Mixing rule

The simplest formula for composite property designated as M_k

$$M_k = \Sigma M_{di} * v_{di} + M_m * v_m$$

- probabilistic calculation principle.

For this property must not pay a synergistic effect.

By volume fractions is

$$v_d + v_m = 1$$

(only one dispersion!)

Usuelly we know v_d and compute

$$v_m = 1 - v_d$$

Density of composite

Mass of composite $m_k = m_m + m_d$,

at the same :

$m_k = \rho_k * V_k$, $m_m = \rho_m * V_m$, $m_d = \rho_d * V_d$

It easily deduce the density

of the composite :

$$\rho_k = \rho_m * v_m + \rho_d * v_d$$

By density we can always apply mixing rule.

That allows light determine from density the percent of dispersion in the composite.

Can be disrupted by the presence of pores

Determining of pores

Quantity of pores in composite we can determine from density.

Because

$$v_m + v_p + v_d = 1$$, is valid :

$$\boldsymbol{\rho_k = \rho_m * v_m + \rho_d * (1 - v_m - v_p)}$$

From that we can compute

$$\boldsymbol{v_p = 1 - \varrho_k / \varrho_d + v_m * (1 - \varrho_m / \varrho_d)}$$

Voigt's model

Assuming longitudinal tensile load of fiber composite.

That is why longitudinal model.

Extension of whole composite, fibers and matrix is the same, je stejné, the same is a relative deformation, so

$$\varepsilon_k = \varepsilon_d = \varepsilon_m$$

The force is distributed between the fibers and matrix, thus

$$P_k = P_d + P_m$$

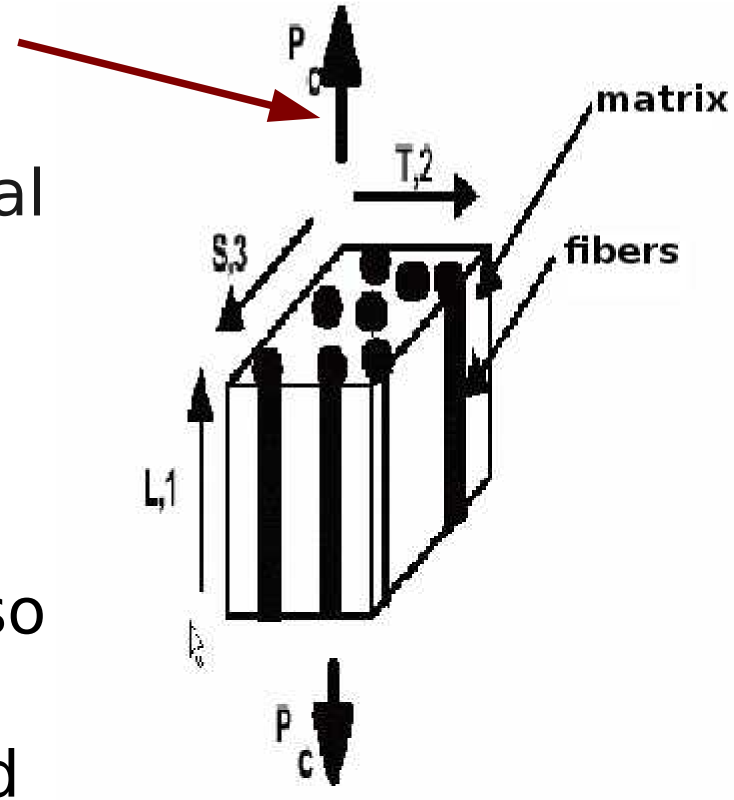
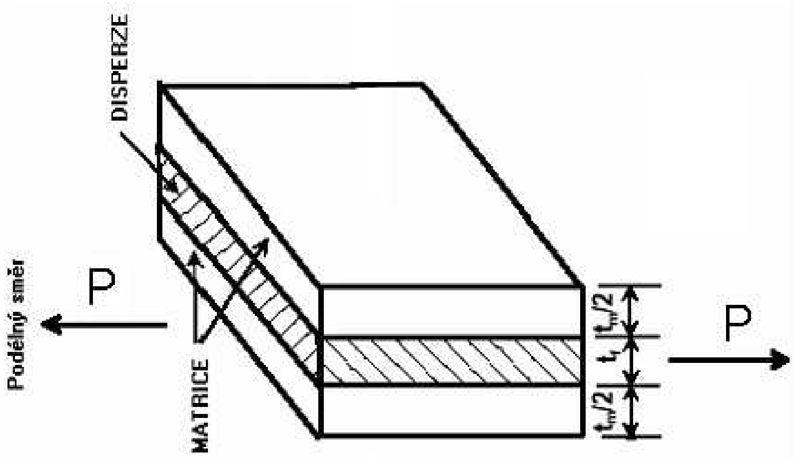


Plate composite



- The same is by plate composite with force paralel to planes.

Compute of forces

In whole composite, as in fibers and matrix load force is the product of voltage and the cross-section.

Expression of load forces with crossections and stress and combination with relation for load force of composite gives us

$$A_k*\sigma_k = A_d*\sigma_d + A_m*\sigma_m$$

Dividing with A_k gives us relation

$$\sigma_k = v_d*\sigma_d + v_m*\sigma_m$$

Combination with Hooke's law

If the load forces are just so small that applies Hooke's law for the fiber and matrix (within the elastic limit) must apply to the composite as a whole proportionality between stress and strain, tension is thus always the product of the relative strain and Young's modulus E.

Substituting Hooke's law to a previous relationship relationship :

$$E_k * \varepsilon_k = v_d * E_d * \varepsilon_d + v_m * E_m * \varepsilon_m$$

The basic relation

The basic assumption of the model which we proceeded, was that the relative deformation of fibers, matrix and composite are the same. Thus, we can divide the previous relationship with relative deformation

$$\varepsilon_k = \varepsilon_d = \varepsilon_m$$

And we obtain the basic relationship

$$\mathbf{E_k = v_d * E_d + v_m * E_m}$$

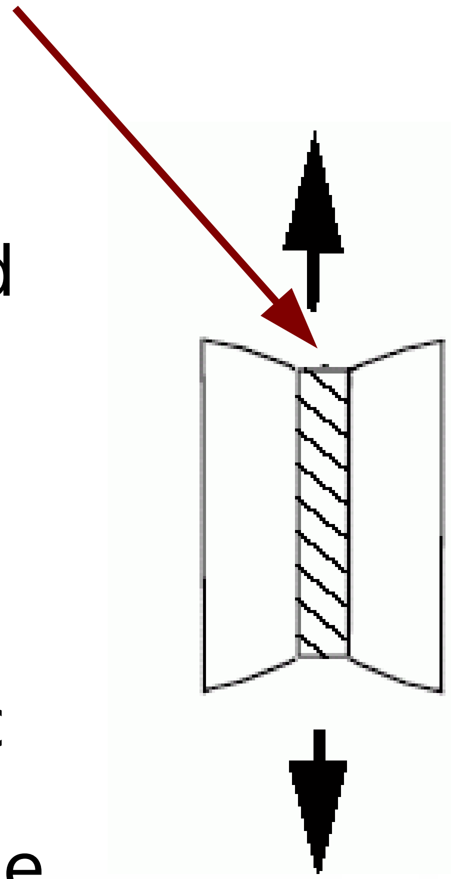
The Young's modulus is valid in Voigt model mixing rule

Considerations of accuracy

The matrix will not deform the entire width between the fibers as well.

Lateral strain – Poisson's number of matrix and dispersion are not the same, the matrix and fibers in the transverse direction will deform differently, so that tension arises in the transverse direction.

It is therefore not entirely accurate Voigt model, the elastic energy considerations show that provides an upper estimate of the Young's modulus.



Consequences of Voigt's model for stress

Therefore deformation of fibers and matrix is the same, we have relation

$$\varepsilon = \sigma_d / E_d = \sigma_m / E_m$$

Relation for ratio of stresses in dispersion and matrix must be

$$E_d / E_m = \sigma_d / \sigma_m$$

To be the most loaded dispersion (assumed the greatest share of the matrix load, and the composite as much to reiforce) must dispersion have a maximum Young's modulus against the modulus of matrix.

Consequences of Voigt's model for loading force

Therefore the force in fibers and in matrix is always product of crossection and stress, is valid

$$P_d / P_m = E_d / E_m * A_d / A_m$$

After dividing whole crossection of composite :

$$P_d / P_m = E_d / E_m * v_d / v_m$$

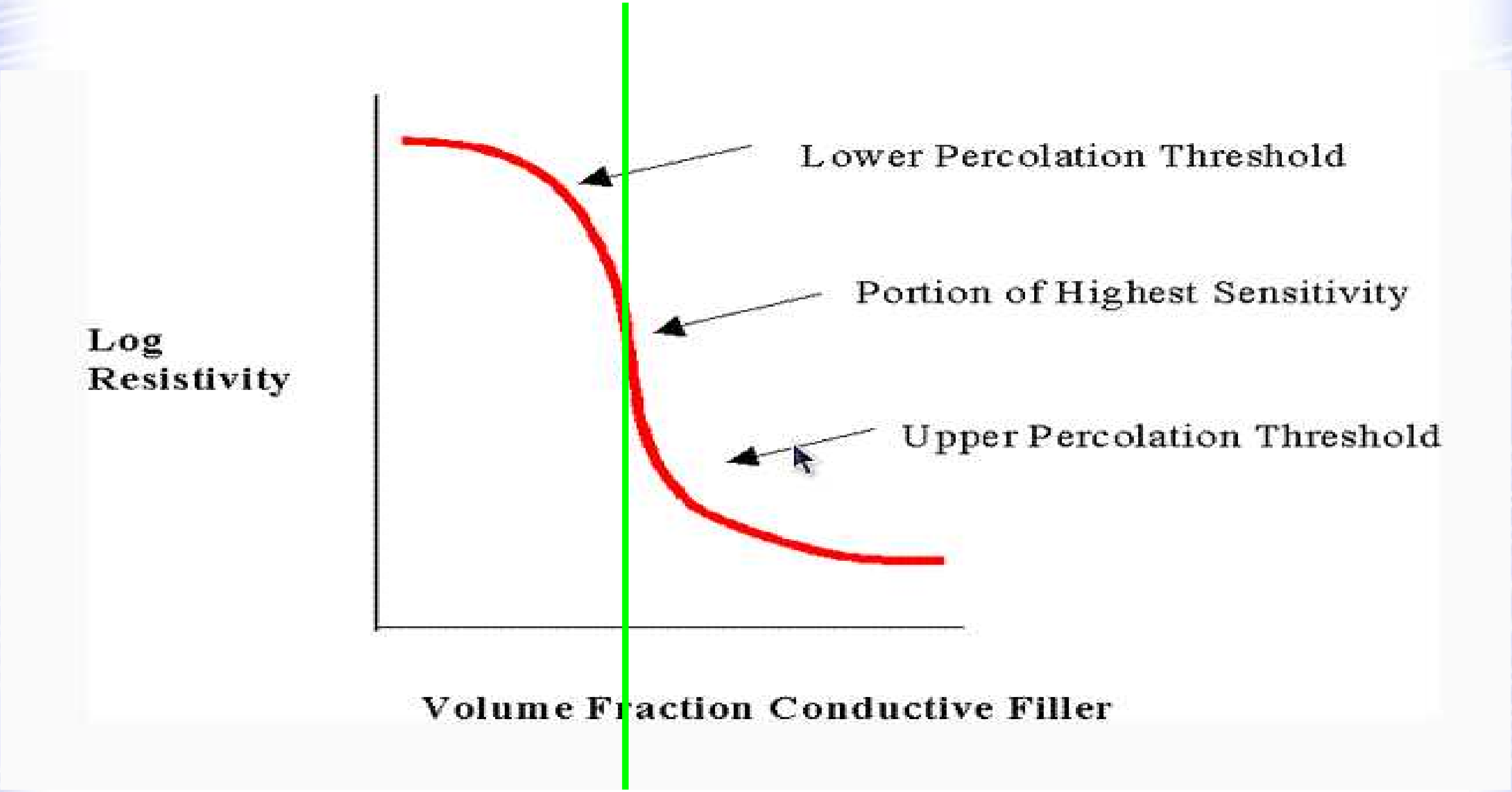
Dispersion to bear the greatest share of the load, it must have a large Young's modulus, and it must also be the most.

Percolation limit

The fibers initially, when there are few, in the composite can not touch each other everywhere in between matrices. If such fibers are electrically conductive, conductive composite is in the direction of the fibers but nonconductive in the perpendicular direction.

If the number of fibers exceeds the so-called **percolation limit**, they begin to touch each other and composite will be conducting in the transverse direction (for more details on particles). The same applies to water absorption or thermal conductivity.

Example of percolation curve



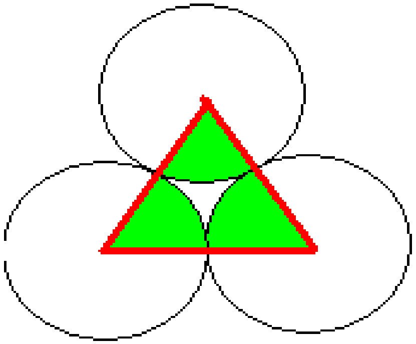
Percolation limit – usually several % of fibers

Maximal number of fibers

Fibers can not come closer to each other than in the adjacent figure - their centers form an equilateral triangle. If the fiber radius r, the area of the fibers within the triangle is just one semicircle, ie $\pi r^2/2$.

Side of the triangle is 2r, high $r * \sqrt{3}$, also its area $r^2 * \sqrt{3}$.

Empty area within triangle (not green) is $r^2 * \sqrt{3} - \pi r^2/2$.



Consequence for composite

Relation of that empty area and area of triangle is :

$(r^2 * \sqrt{3} - \pi r^2/2) / r^2 * \sqrt{3} = 0{,}09$.

If there are in composite above 9 % of matrix (and less then 91 % of fibers), can't in composite (ideally) form pores.

By less then 9 % of matrix begin to form pores - capillary.

In fact, it is not possible to hold fibers as perfectly as we expected - maximal quantity of fiber in composite without pores is about 80%.