Optical and electro-optical properties of crystals I

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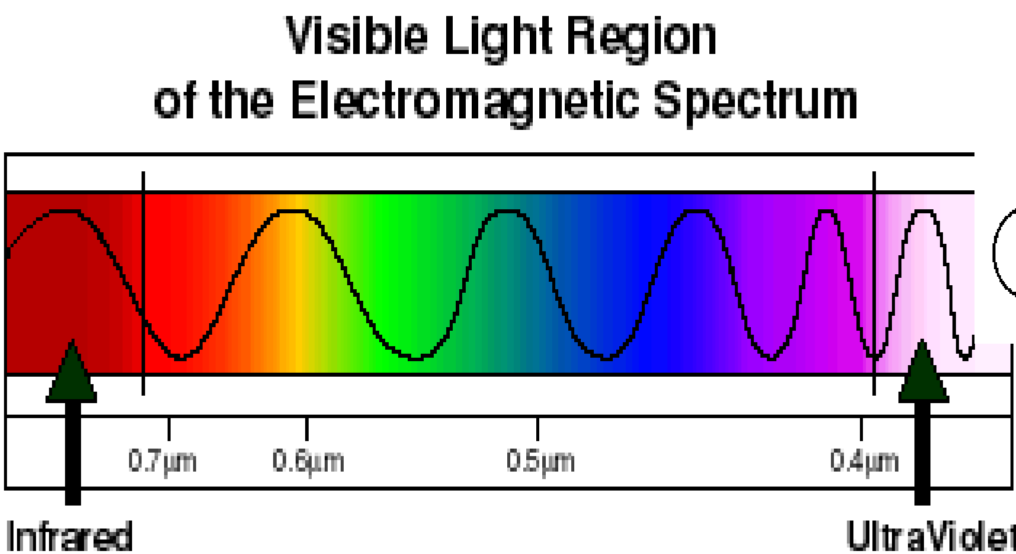
How can optics help to piezoelectric material research

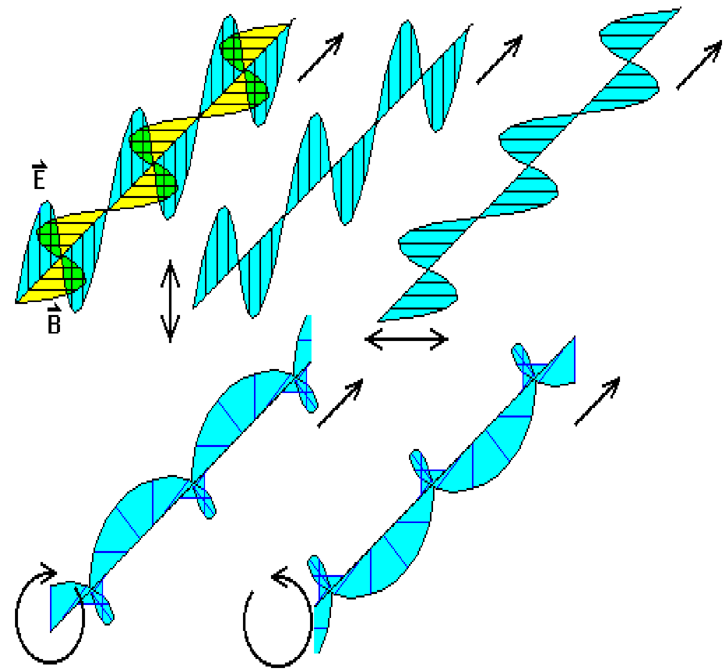
- What are light properties?
- How it can interact with transparent crystals?
  - Optical indicatrix
  - Birefringence
  - Optical activity
- What we can see with polarized microscope?
- How can electric field change indicatrix and what will happen?

Properties of light

Wavelength λ

for visible light λ is ranging from 380 to 760 nm

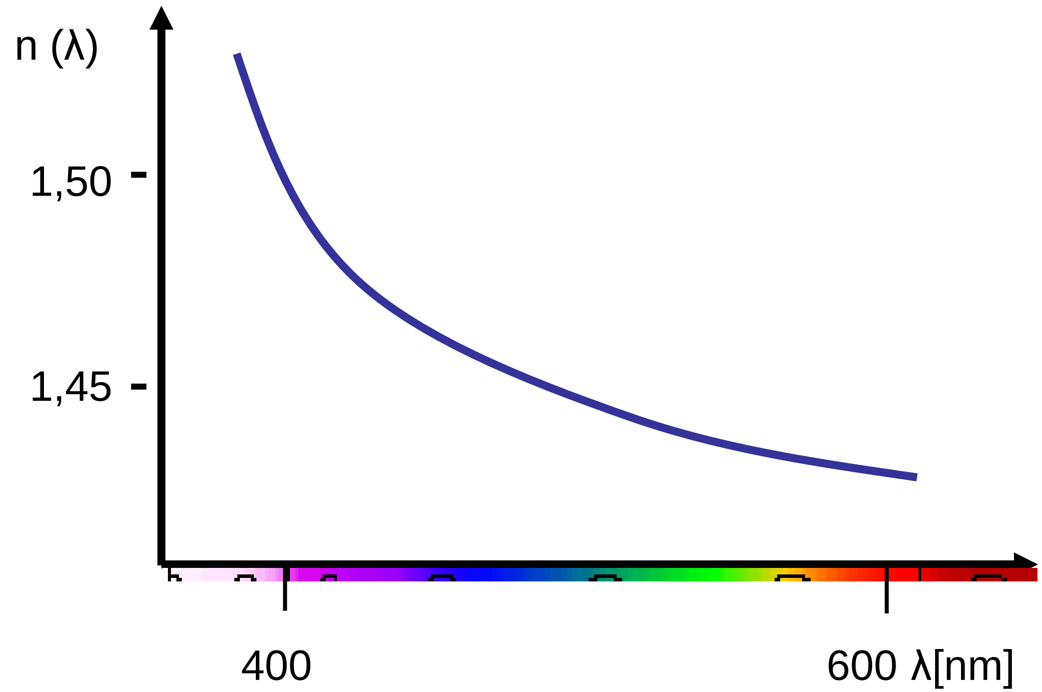




Polarization
- linear
- circular

Interaction of light with materials (without absorption) is described by index of refraction n

$$n(\lambda) = \frac{c}{v(\lambda)}$$



The Optical Indicatrix

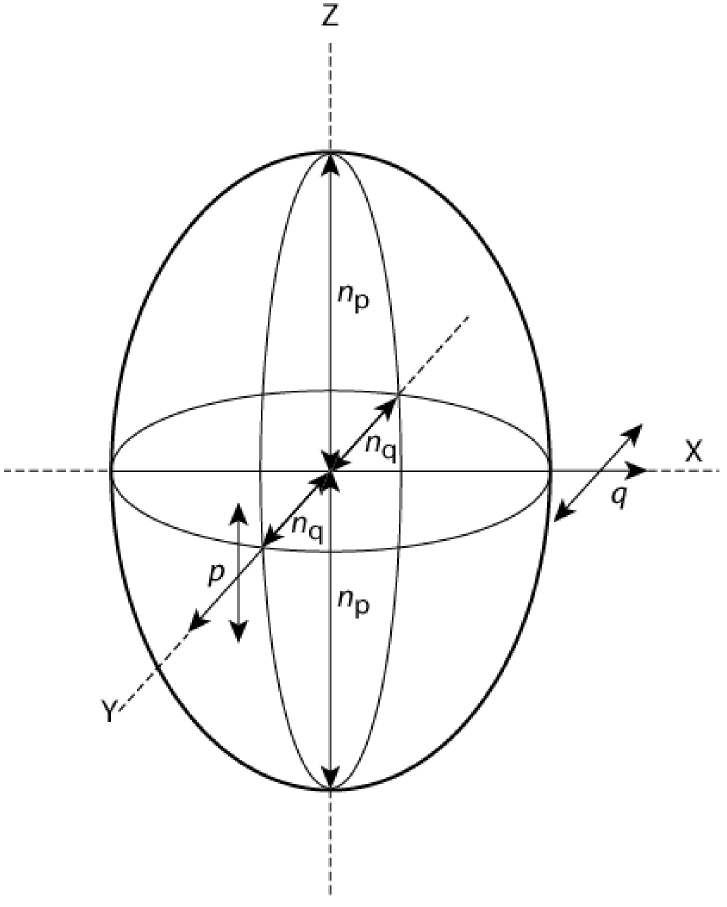
The optical indicatrix describes the index of refraction *n* of linear polarized light variations according to the direction of light passing through a material and **E** vibration direction

$$D_i = \sum_j \varepsilon_{ij} E_j$$

- *n* is connected with dielectric constant. In principal axes system are principal refractive indices

$$n_1 = \sqrt{\varepsilon_{11}/\varepsilon_0} \quad n_2 = \sqrt{\varepsilon_{22}/\varepsilon_0} \quad n_3 = \sqrt{\varepsilon_{33}/\varepsilon_0}$$

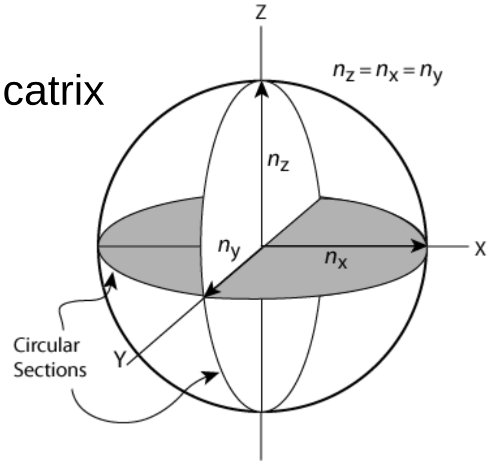
- The indicatrix is constructed by plotting indices of refraction as radii parallel to the vibration direction of the light.



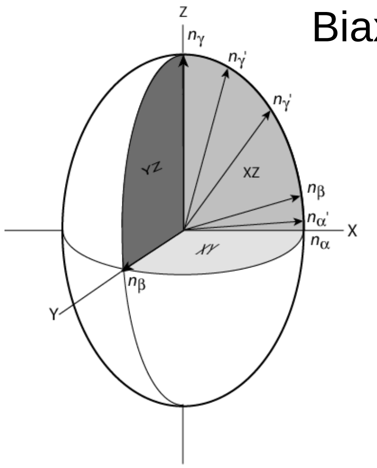
Types of indicatrix

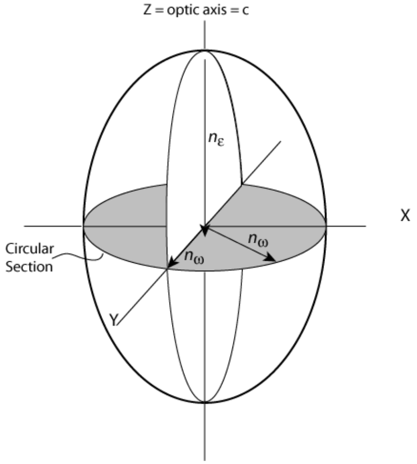
http://micro.magnet.fsu.edu/primer/java/polarizedlight/ellipsoid/index.html

Isotropic Indicatrix

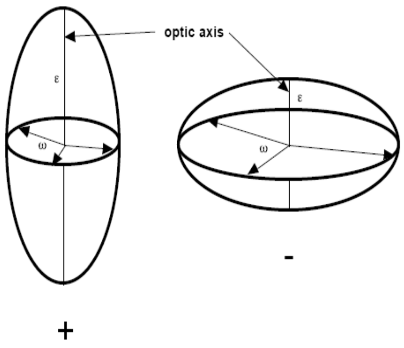


Biaxial Indicatrix





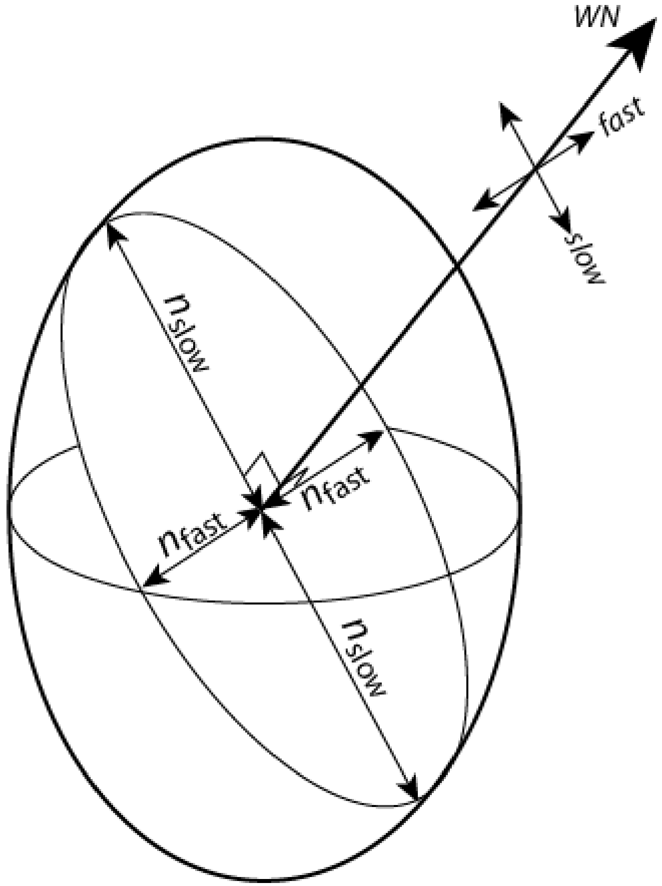
Uniaxial
Indicatrix
+ sign
- sign



Indicatrix and crystalline system

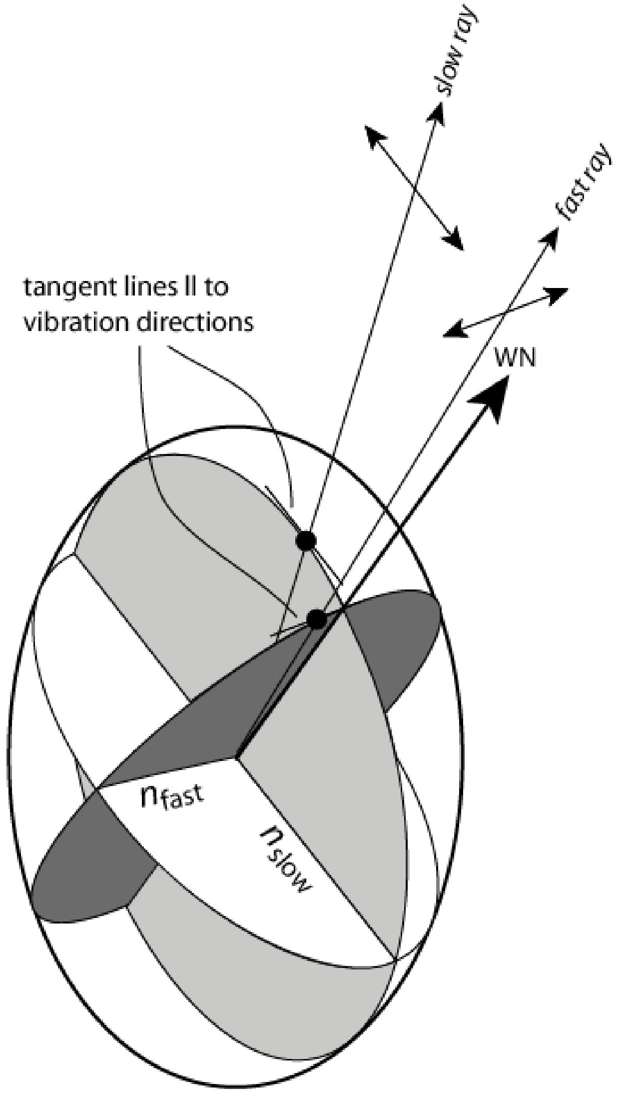
Crystalline system	Linear anisotropy	Crystal	Permittivity tensor	Indicatrix
Triclinic	Biaxial	Mica	$\begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$	Ellipsoid
Monoclinic				
Orthorhombic		alexandrite		
Tetragonal	Uniaxial	BaTiO3, ADP, KDP, TeO2, TiO2	$\begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_x & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$	Ellipsoid of revolution around optical axis
Trigonal		Quartz, LaTaO3, LiNbO3, calcite		
Hexagonal		ZnO, CdS, BaAl2O4		
Cubic	Isotropic	BGO, BSO,…	$\begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}$	Sphere

Optical Indicatrix and direction of wave normal

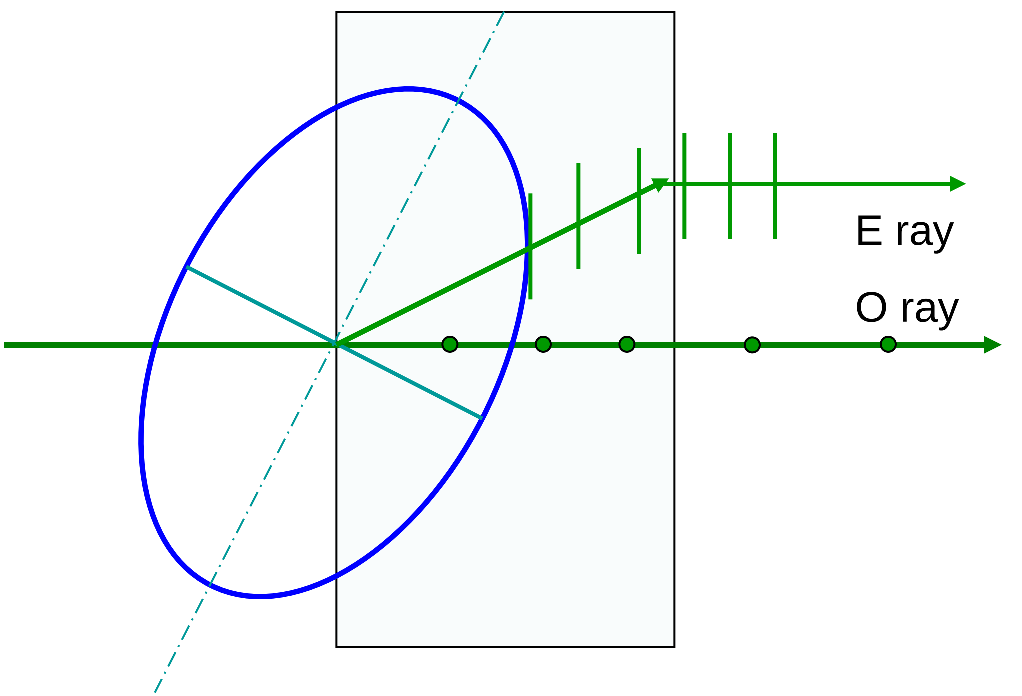


- The sections, parallel to the wave front through the indicatrix, are ellipses (WN is wave-normal)
- The long (short) axis of this elliptical section is parallel to the slow (fast) ray vibration direction and the radius parallel to this direction is equal to the slow ray index of refraction (n_{slow}, n_{fast}).

Ray Paths for Optical Indicatrix

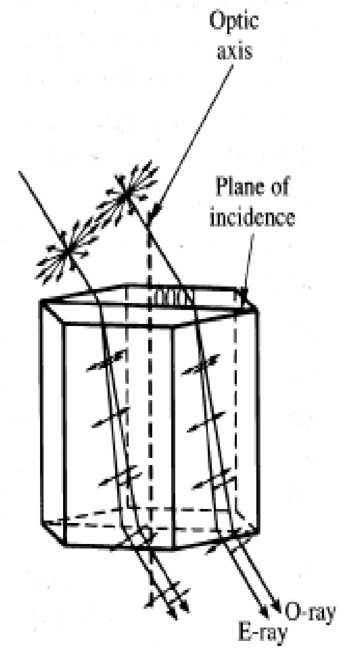


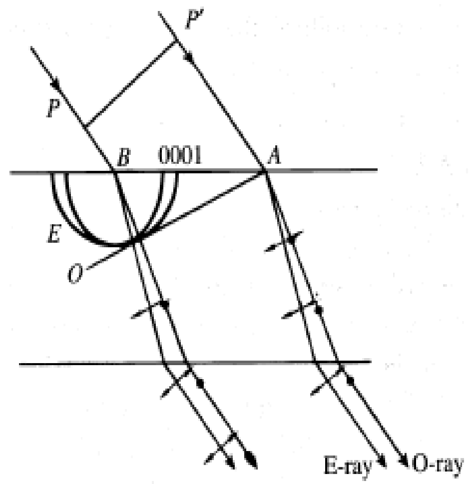
- To find the ray paths, which are paths followed by an image through the mineral, tangents to the indicatrix are constructed parallel to the vibration directions of the slow and the fast rays.



Ordinary and Extraordinary Rays

- The ray, whose waves vibrate in the basal plane is called the ordinary ray whilst the ray whose waves vibrate in a principal section is called the extraordinary ray.
- both rays diverge from their associated wave normals
- Snell's Law is valid only for ordinary ray
- To determine whether the extraordinary ray is the fast or the slow ray we need to know the optic sign of the mineral.

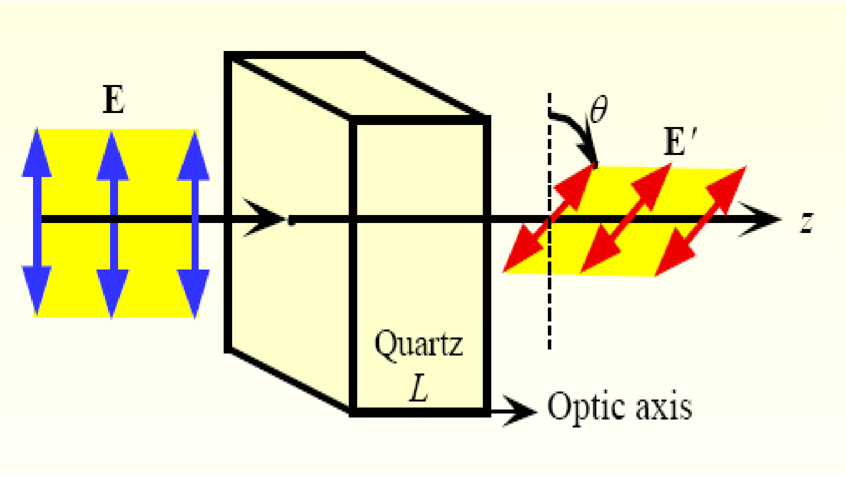


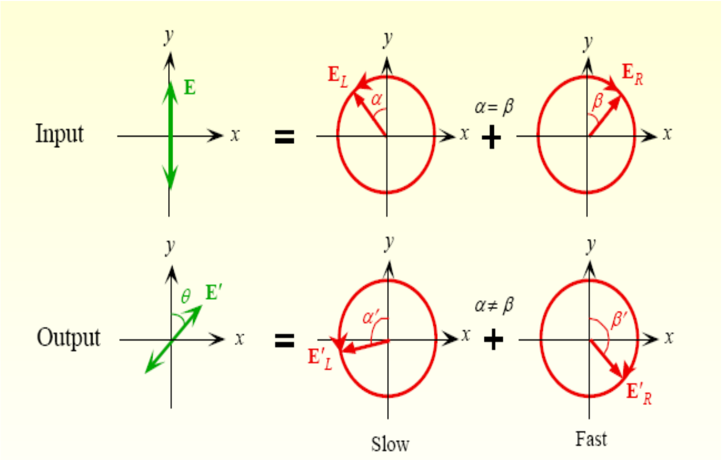


BIREFRINGENCE

Optical Activity and Circular Birefringence

- In some materials (quartz, TeO_2) are different velocities for propagation of circular polarized light clockwise and counterclockwise and two indexes of refraction n_+ and n_-



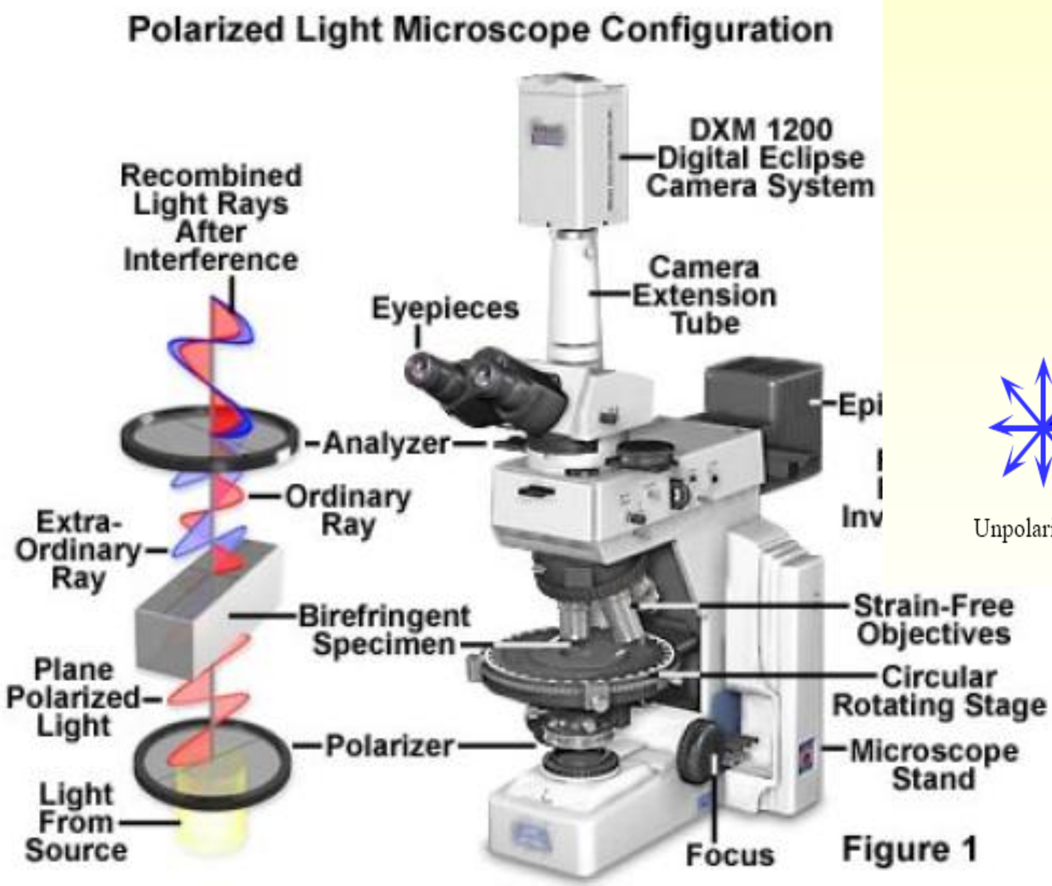


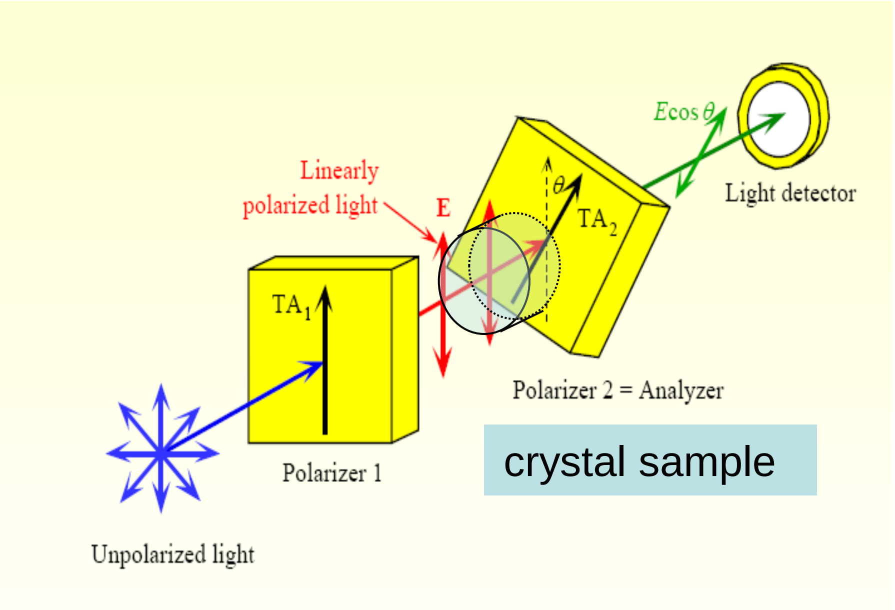
- As a result of optical activity linearly polarized light (which is superposition of two circulated light) rotates plane of polarization.
- This effect is depending on wavelength

Sense of rotation is the same as sense of rotation of faster mode

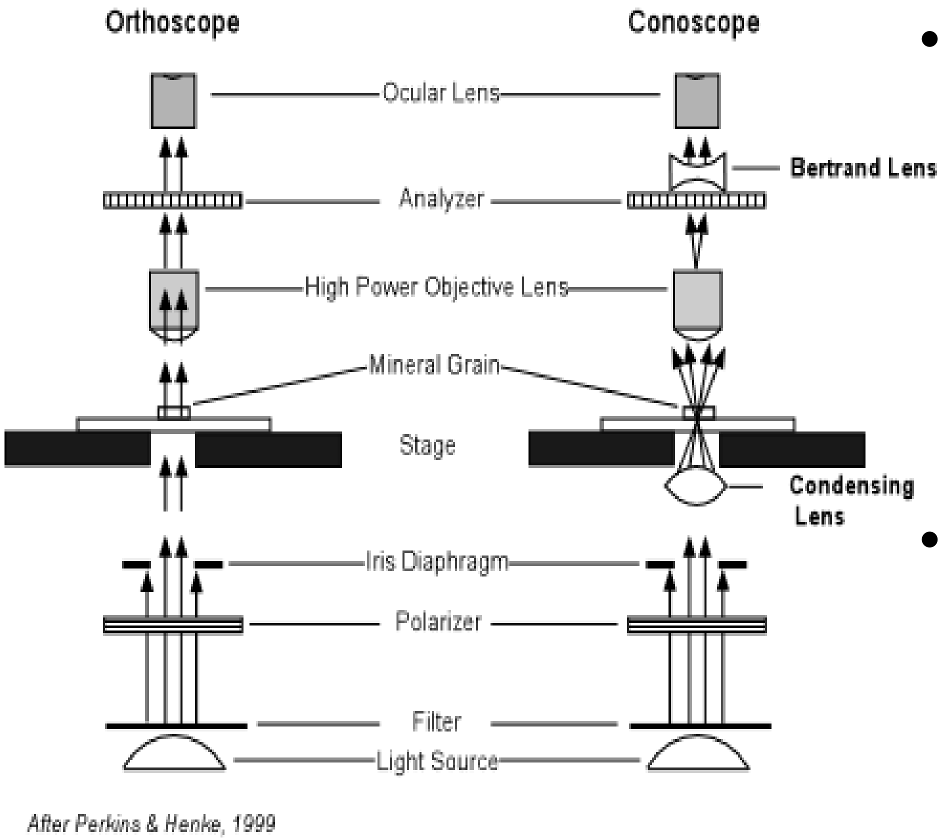
$$\theta = \frac{\pi\left(n_{-} - n_{+}\right)}{\lambda_0} L$$

Polarized microscope for crystal studies



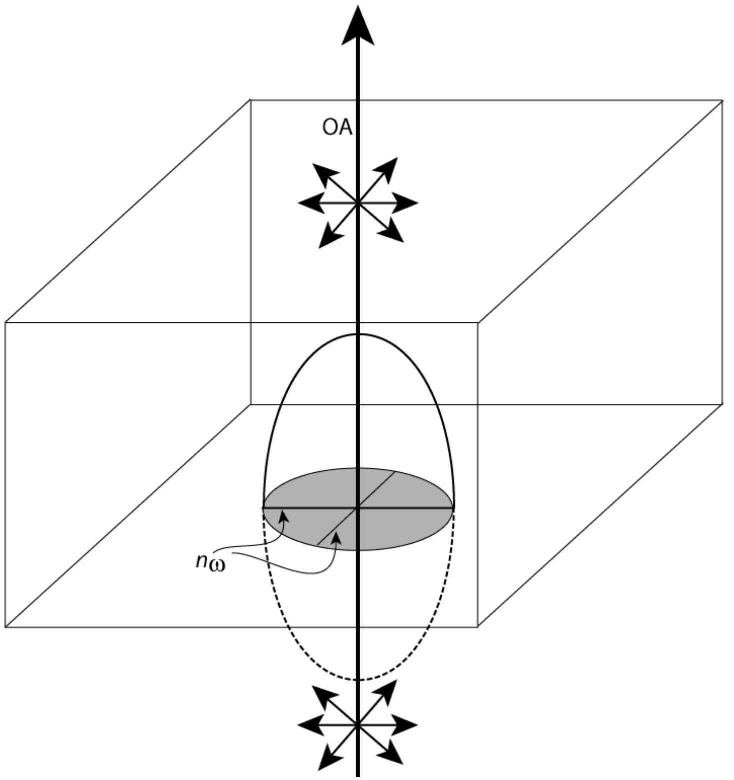


Two possible types of observation

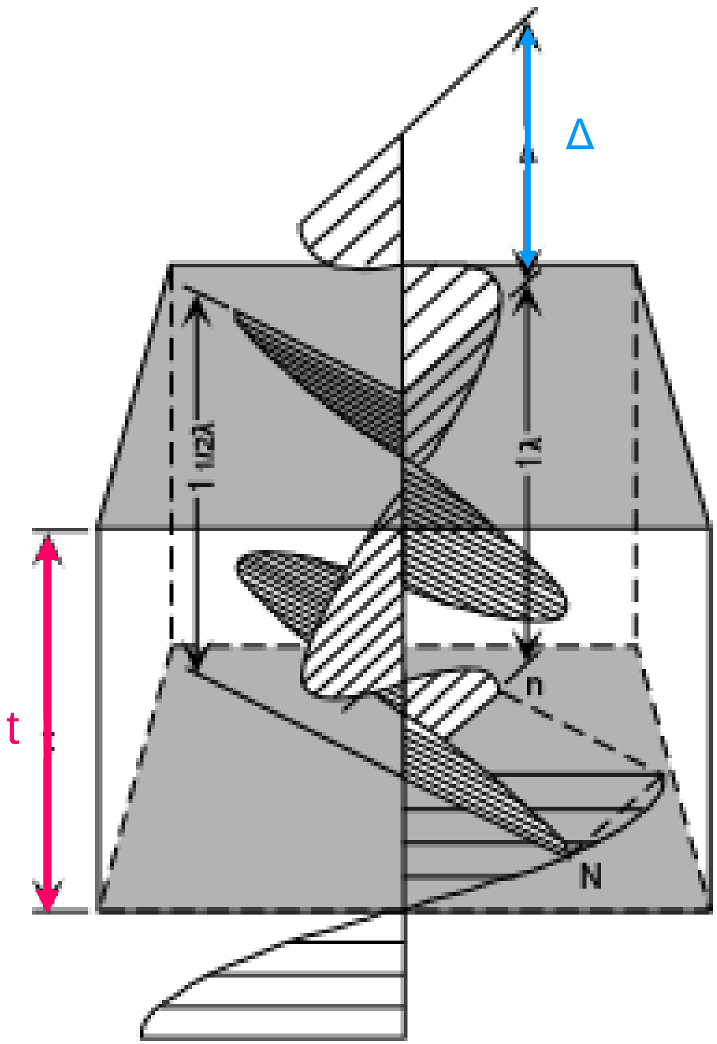


- Orthoscope
  - parallel rays pass through crystal strike the bottom surface of the sample perpendicular to the surface, the section through the indicatrix are the same for all rays
- Conoscope
  - convergent rays pass through crystal and cross indicatrix in different cross-sections, so every ray has his section and n_{ε}

Orthoscope - Uniaxial Indicatrix



- For vertical optical axis (parallel to ray) this crystal should behave like an isotropic mineral between crossed polarizers and remain dark as the stage with sample is rotated.



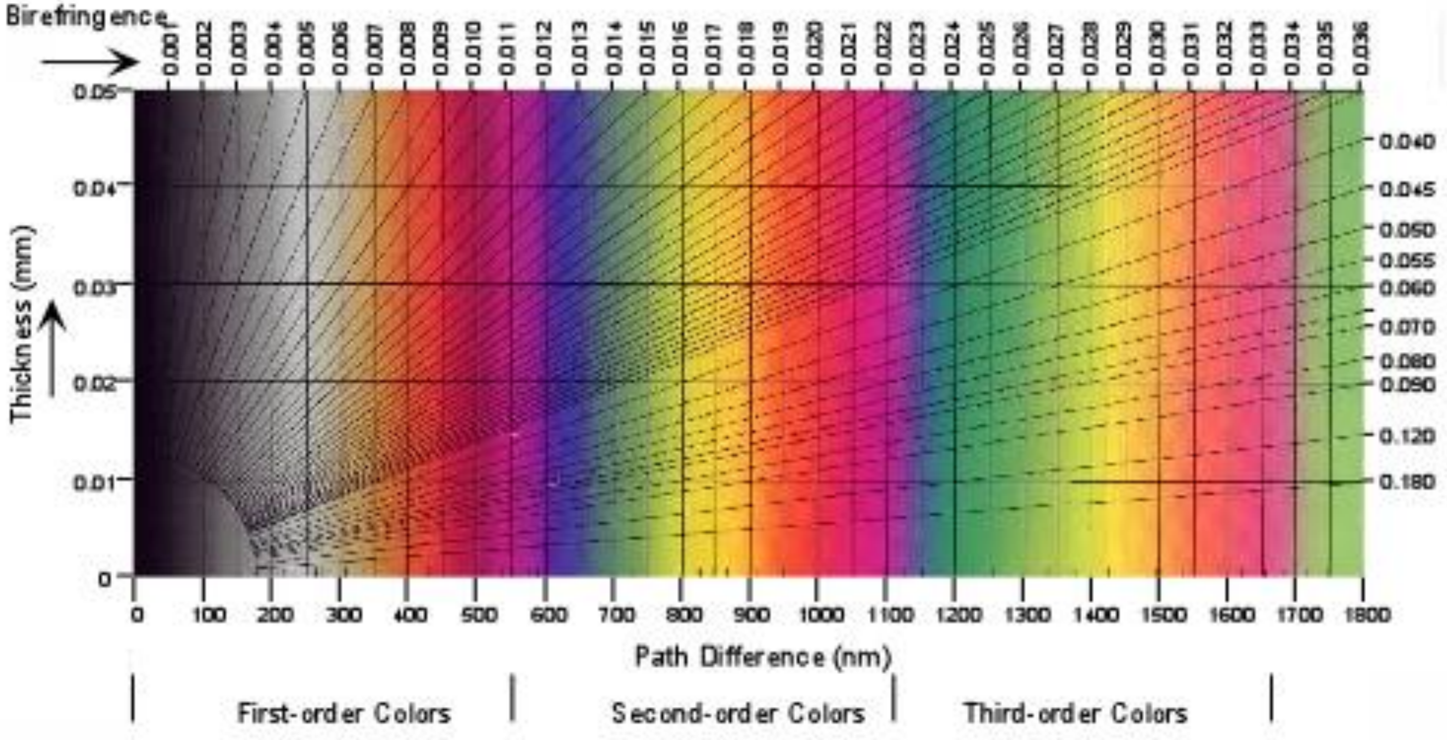
- If the crystal is oriented in a random orientation the section through the indicatrix is an ellipse whose axes are n_ω and $n_{\varepsilon'}$.

- Two orthogonal polarization of light have different velocities so some retardation Δ of this ones is observed in sample of thickness t. The light from crystal will be elliptical polarized.

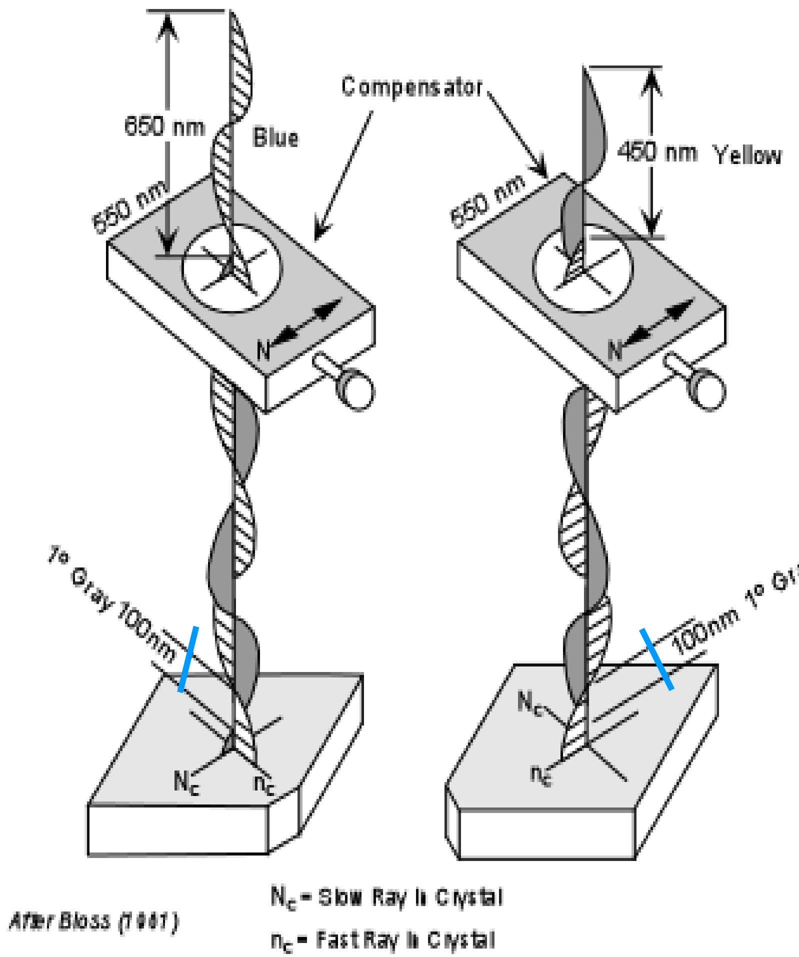
$$\Delta = t\,(n_{slow} - n_{fast})$$

Interference colors

- Due a dispersion phase retardation depends on wavelength. Some wavelengths (colors) can obtain 0, 2π, 4 π retardation … so original linear polarization is conserved and this color disappears between crossed polarizers. Some wavelengths has a retardation π, 3 π so linear polarization is rotated about π/2. This color will be enhanced between crossed polarizers.
- If the crystal is oriented so that its optic axis is perpendicular to rays, birefringence and hence interference colors are maximum values.

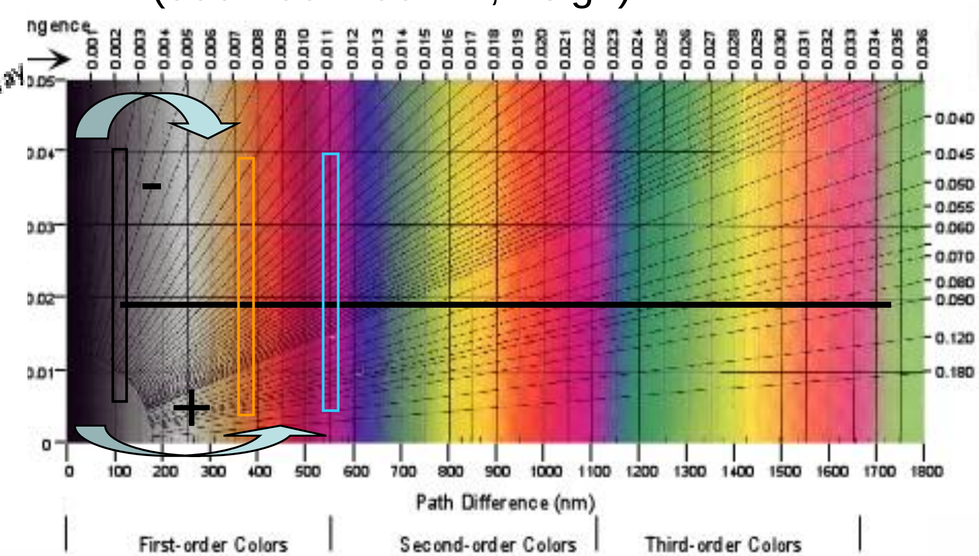


Determination if crystal is positive or negative

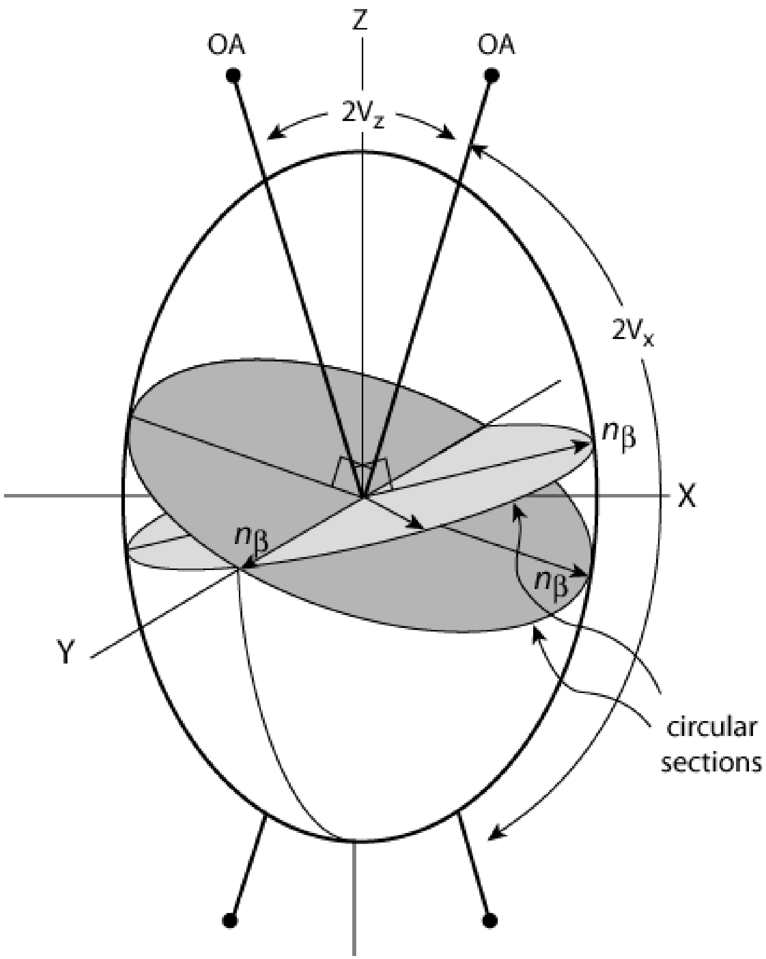


- The dimensions of the indicatrix along the c axis may be either greater or less than the dimensions at right angles.
- optically positive minerals, $n\varepsilon > n\omega$

  optically negative minerals, $n\varepsilon < n\omega$
- In orthoscope observation is possible to add wavelength compensator (retardation 2π for 550 nm) and shift interference for example from gray to blue (550+100=650, + sign) or yellow (550-100=450 nm, - sign)



Orthoscope - Biaxial indicatrix

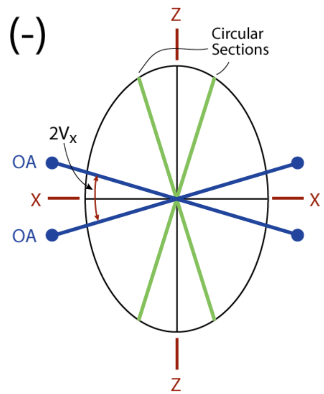


There are 2 different ways to cut this indicatrix to get a circle…

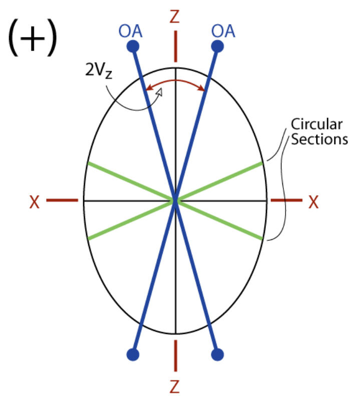
the 2 optic axes are perpendicular to the circular sections in biaxial minerals

The angle between the optic axes bisected by the X axis is also called the 2Vx angle, while the angle between the optic axes bisected by the Z axis is called the 2Vz angle where 2Vx + 2Vz = 180°.

Optic Sign of Biaxial Indicatrix



Biaxial Negative
Acute Bisectrix (Bxa) = X
Obtuse Bisectrix (Bxo) = Z



Biaxial Positive
Acute Bisectrix (Bxa) = Z
Obtuse Bisectrix (Bxo) = X

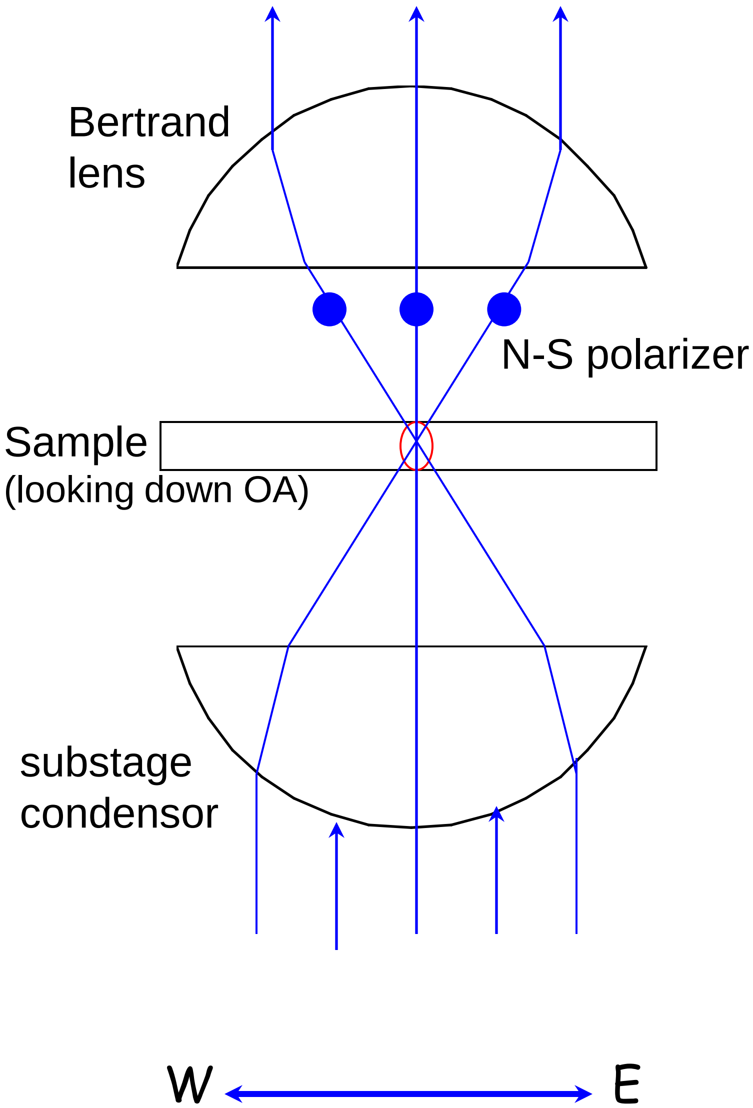
The optic sign of biaxial minerals depends on whether the Z or Z indicatrix axis bisects the acute angle between the optic axes.

- When 2V is acute about Z: sign (+)
- When 2V is acute about X: sign (-)
- When 2V=90°, sign is indeterminate
- When 2V=0°, mineral is uniaxial

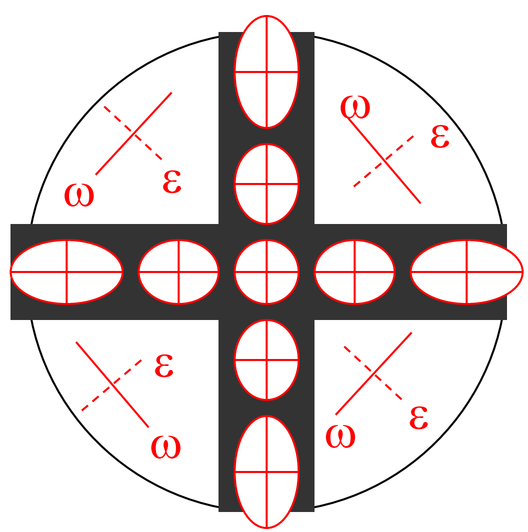
Orthoscope observations of the Biaxial Indicatrix crystals

- The biaxial indicatrix is used in the same way as the uniaxial indicatrix.
- It provides information about the indices of refraction and vibration direction given the wave normal direction that light is following through a mineral.
- Birefringence depends on how the sample is cut.
- Birefringence is:
  - A maximum if the optic normal is vertical
  - A minimum if an optic axis is vertical
  - Intermediate for random orientations

Conoscope unaxial example

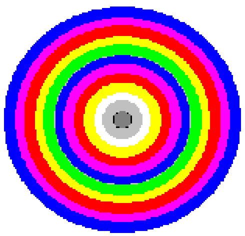


Converging lenses force light rays to follow different paths through the indicatrix

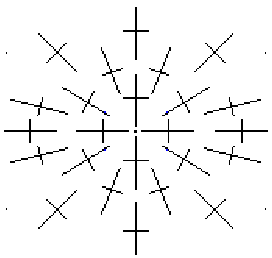


Effects of multiple cuts thru indicatrix

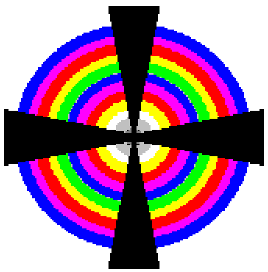
Conoscope observation of unaxial crystals



Concentric circular isochromes



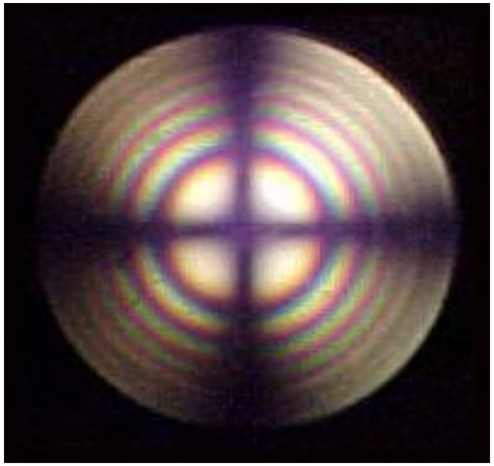
Radial and Tangential polarization directions



Resulting Interference Pattern

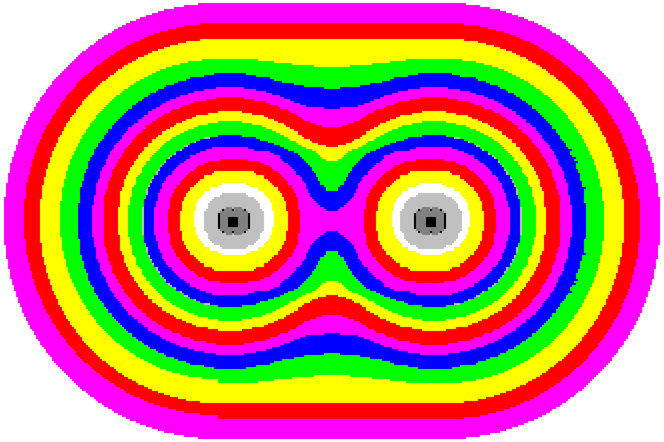
Isogyres - areas of extinction

Isochromes – lines of equal retardation.



- Viewed along the optic axis, the indicatrix of a uniaxial mineral has a circular cross-section. Birefringence and thickness both increase uniformly with increasing angle from the optic axis. The isochromes are circular.
- The polarization directions of the rays are radial and tangential to the optic axis. The isogyre is a black cross superimposed on the isochromes, with its arms parallel to the polarizer directions.

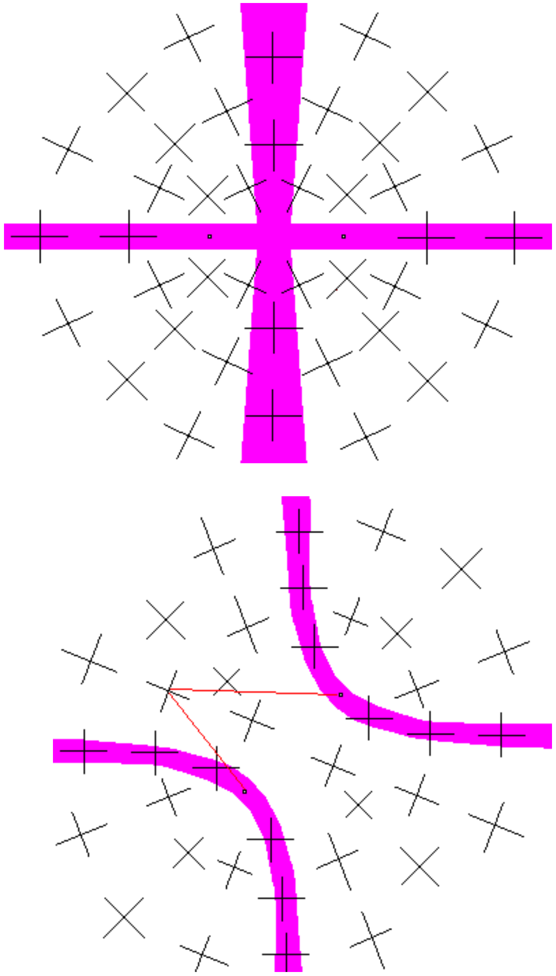
Isochromes in biaxial crystal



- There are two optic axis directions. The plane containing the optic axes is the *optic axial plane.* Close to each optic axis, you see concentric isochromes. Farther away the isochromes loop around both axes

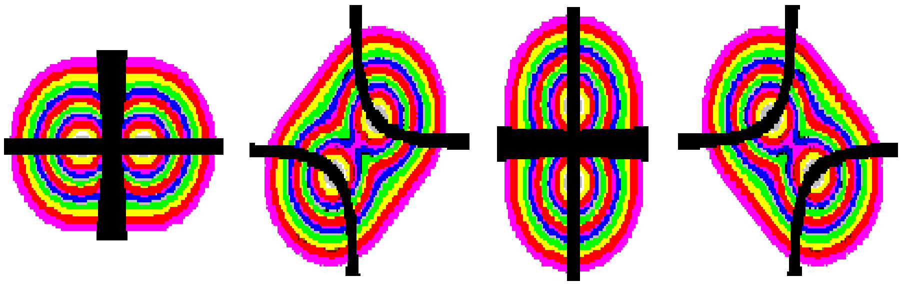
Isochromes move with quarter-wavelength plate insertion.
The sign of crystal is determined similarly to uniaxial crystal

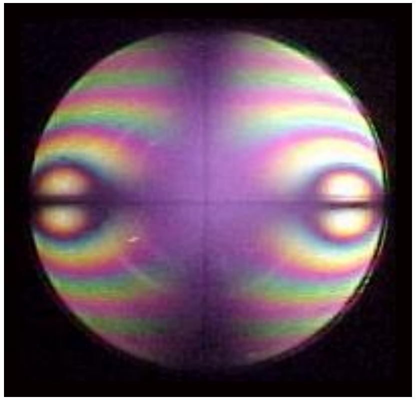
Isogyres in biaxial crystal

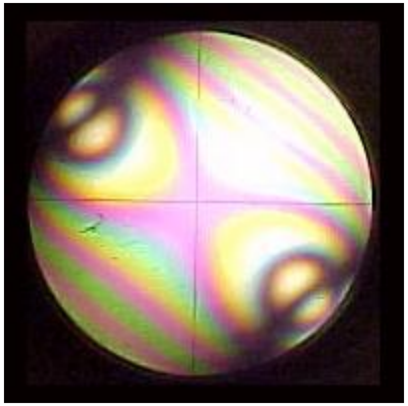


- When the optic axial plane lies on one of the polarizer directions, the isogyre is a black cross. It differs a bit from the uniaxial case: the arm perpendicular to the optic axial plane is thicker.
- When the optic axial plane is at 45 degrees to the polarizers, the isogyre is two hyperbolas in opposite quadrants asymptotic to the polarizer direction and passing through the optic axes.

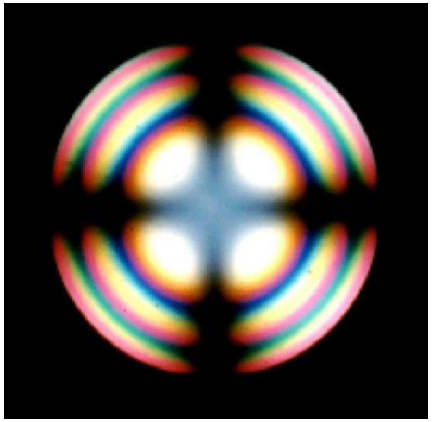
As the grain rotates, the cross splits into two hyperbolas that diverge, then reconverge and merge again into a cross. They then diverge into the other pair of quadrants, reconverge, and so on. The distance the hyperbolas separate is a measure of 2V, the angle between the optic axes. 2V can also be estimated for large angles by how quickly the hyperbolas leave the field of view. The larger 2V, the faster the hyperbolas disappear. Each microscope will behave a bit differently, so it's necessary to observe known materials first to calibrate the observations.





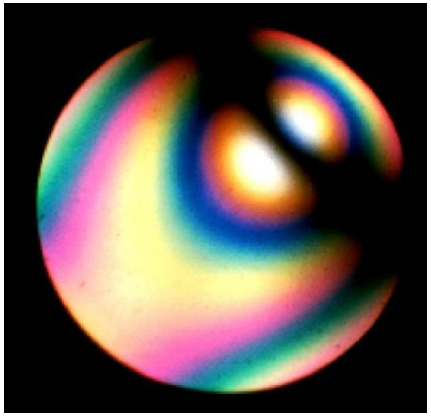


Determining if mineral is uniaxial or biaxial

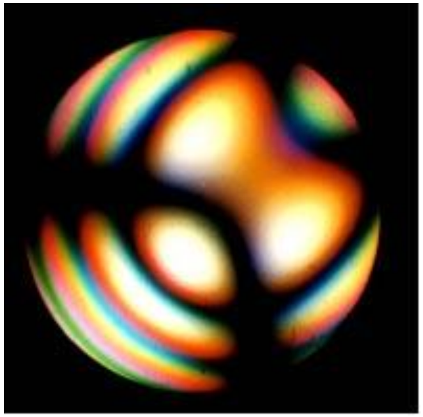


uniaxial

If uniaxial, isogyres define cross; arms remain N-S/E-W as stage is rotated



or



biaxial

If biaxial, isogyres define curve that rotates with stage, or cross that breaks up as stage is rotated

Conoscopis observation
Find a grain that stays **dark** as stage is rotated
Go to highest power objective
Insert Bertrand Lens
Look down scope and rotate stage

determine 2V from curvature of isogyre